Gainesville College – Tenth Annual Mathematics Tournament

You may write in this test booklet. Only the elect

4. Let $f(x)$ be a function such that the graph of $f(x)$ is a semicircle with end points (a,0) and (b,0) where $a < b$. Find: $\left| \int_a^b f(x)\,dx \right|$

90 feet

3rd Base

90 feet

11. Solve the differential equation $\frac{dy}{dx} = \frac{2x}{x^2 - 9}$, finding the solution which passes through the point (0,4).

a) $y = \ln\left|x^2 - 9\right| + 4 - \ln 9$

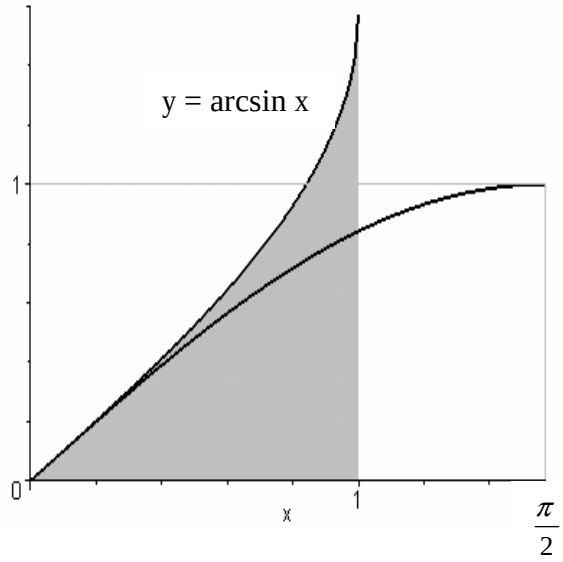
b) $y = \ln\left|x^2 - 9\right| - 4 + \ln 9$

c) $y = \ln\left|x^2 - 9\right| - 4 - \ln 9$

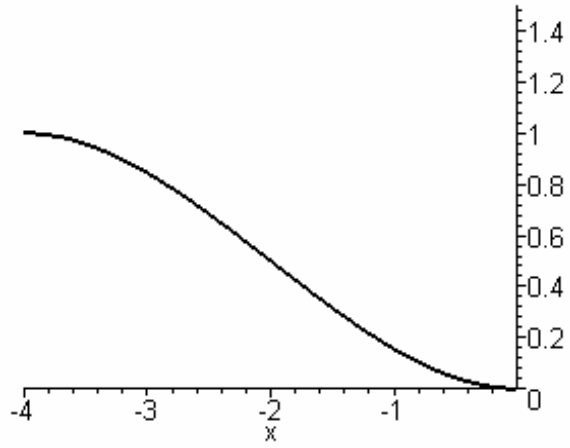
d) $y = \ln\left|x^2 - 9\right| + 4 + \ln 9$

e) none of the above

12. Use the accompanying figure to calculate: $\int_0^{\pi/2} \sin x \, dx$ $\int_0^1 \arcsin$



13. A small aircraft is flying horizontally until it starts its descent from an altitude of 1 mile, 4 miles west of the runway. Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ on the interval [-4,0] that describes a smooth glide path for the landing, as modeled by the figure below.



a) $f(x) = \frac{1}{32}x^3 + \frac{5}{16}x^2 + \frac{1}{2}x$

b) $f(x) = \frac{1}{16}x^3 + \frac{5}{16}x^2$

c) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$

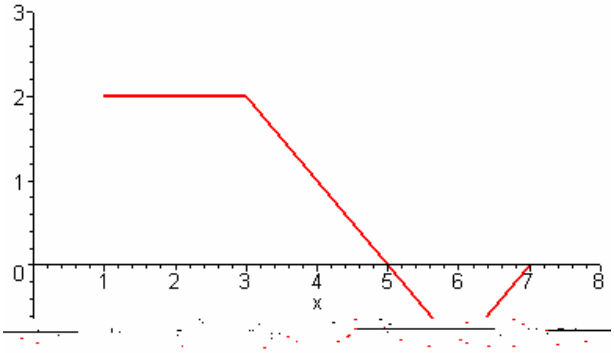
d) $f(x) = \frac{1}{32}x^3 - \frac{3}{4}x^2$

e) none of the above

14. Let $x\sin \pi x = \int_0^{x^2} f(t)\,dt$. Find $f(4)$.

a) 16π
b) 0
c) 4
d) $\frac{\pi}{2}$
e) none of the above

15. The graph of f is given in the figure below. Determine the average value of f on the interval [1,7].



a) $\frac{7}{6}$
b) $\frac{5}{6}$
c) 1
d) $\frac{2}{3}$
e) none of the above

19. Find: $\int_0^{\infty}\left(1-\frac{e^x}{2+e^x}\right)dx$

a) 0
b) $\ln 2$
c) $\ln 3$
d) ∞
e) none of the above

20. Find the area of the following region: $\begin{cases} y \geq x^2 \\ x^2 + y^2 \leq 2 \end{cases}$

a) $\frac{1}{3}+\frac{\pi}{2}$
b) $\frac{\pi}{2}$
c) $2\pi-\frac{4}{3}$
d) $\frac{4}{3}$
e) none of the above

21. Given $f(x) = x^2e^{-x^2}$, find all values of x

22. Given $\int_1^3 f(x)\,dx = 4$ and $\int_1^3 g(x)\,dx = 2$, find $\int_1^3 \left[2f(x)+5g(x)\right]dx$.

a) 54
b) 13
c) 24
d) 18
e) none of the above

23. Assume that f is differentiable for all x. The sign of f' is as follows:

$f'(x) > 0$ on $(-\infty,-4)$
$f'(x) < 0$ on $(-4,6)$
$f'(x) > 0$ on $(6,\infty)$

Let $g(x) = f(10-2x)$. Then $g'(5)$ is

a) positive
b) negative
c) zero
d) The function g is not differentiable at $x = 5$.
e) none of the above

24. Let $f(x) = \cos(2x)$. Find $f^{(2004)}(0)$, where $f^{(n)}(x)$ denotes the n^{th} derivative of $f(x)$.

a) -2^{2004}
b) 2^{2004}
c) 0
d) 1
e) none of the above

25. Which of the following is true of the behavior of $f(x)=\dfrac{x^3+8}{x^2-4}$ as $x \to 2$?

a) The limit is 0.
b) The limit is 1.
c) The left-hand and the right-hand limits are finite, but not equal.
d) The graph of the function has a vertical asymptote at $x = 2$.
e) none of the above

26. Find: $\int_0^1 (x \ln x)\, dx$

a) -1
b) $-\dfrac{1}{2}$
c) $-\dfrac{1}{4}$
d) 1
e) none of the above

27. Let $f(x)=\begin{cases} 0, & \text{if } x \text{ is rational} \\ 5x, & \text{if } x \text{ is irrational} \end{cases}$. Then

a) f is continuous at every rational number.
b) f is continuous at every irrational number.
c) f is discontinuous everywhere.
d) f is continuous only at x = 0.
e) none of the above

28. An entrepreneur is told that to manufacture a newly designed bike, the expenses break down as follows:

30 dollars per bike in parts
20 dollars per bike in labor
10,000 dollars per day in fixed plant costs

Further, she is told that with the conditions of the plant and the nature of the workers, manufacturing 100 bicycles a day should optimize the usefulness of the equipment and the employees. If more or less than 100 bicycles are manufactured, she should expect additional costs of 2 dollars times the squared difference between the number manufactured and 100. To minimize the average cost per bike, how many should she have manufactured each day?

a) fewer than 100
b) exactly 100
c) more than 100 but not more than 110
d) more than 110 but not more than 120
e) more than 120

29. Suppose f and g are differentiable, and that $(\quad)^2 (\quad)$

31. Let $( )$ $\frac{\sqrt{\ ^{2}}}{2}$

34. If $\int_0^b \tan x \, dx = 2$, then b could equal

a) $\arccos(2)$

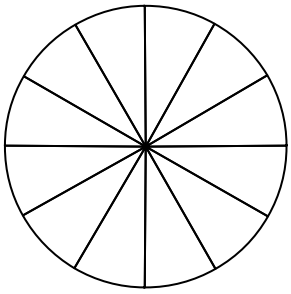
b) $\arccos(2e)$

c) $\text{arcsec}(2)$

d) $\text{arcsec}^2(e)$

e) none of the above

37. Ще



40. For $f(x) = \sin\left[\cos\left(2x - \frac{\pi}{2}\right)\right]$, find $f'\left(-\frac{\pi}{4}\right)$.

a) -1
b) 1
c) -2
d) 0
e) none of the above