
LEADING THE WAY

3. Let $f(x)$ be a one-to-one continuous function such that $f(1)=4$ and $f(6)=2$. Assume $\int_1^6 f(x)\,dx=15$. Calculate $\int_2^4 f^{-1}(x)\,dx$

7. The length of a rectangle is 16 inches and the width is 13 inches. If the area A is increasing at $1\ \text{in}^2/\text{min}$, at what rate must the width be changing so that the length is increasing at $5\ \text{in}/\text{min}$?

a) $-4\ \text{in}/\text{min}$
b) $4\ \text{in}/\text{min}$
c) $\frac{1}{5}\ \text{in}/\text{min}$
d) $-\frac{1}{5}\ \text{in}/\text{min}$
e) none of the above

8. Find the maximum value of the function $f(x) = -\left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right)$.

a) -2
b) 0
c) -1
d) $-e$
e) none of the above

9. Find $\frac{d^2y}{dx^2}$ if $x^2 - y^4 = 6$.

a) $\frac{-3}{16y^7}$
b) $\frac{y}{3}$

10. Find $\lim_{y\to 2}\left[\frac{1}{y-2}\left(\frac{1}{x+y-2}-\frac{1}{x}\right)\right]$.

a) 0
b) $\ln x$
c) $-\frac{1}{x^2}$
d) ∞
e) none of the above

11. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\,dx$.

a) 0
b) 1
c) 2
d) 3
e) none of the above

12. Find the differential dy of $y=\frac{e}{e^x}$ when $x=1$.

a) $1\,dx$
b) $0\,dx$
c) $-1\,dx$
d) $e\,dx$
e) none of the above

13. Tell where the function given by $f(x)=\sqrt{\frac{1-x^2}{4-x^2}}$ is continuous.

a) $[-1,1]$
b) $(-\infty,-2)\cup(2,\infty)$
c) $(-\infty,-2)\cup(-1,1)\cup(2,\infty)$
d) $(-\infty,-2)\cup[-1,1]\cup(2,\infty)$
e) none of the above

14. Find the volume V of the solid of revolution formed by revolving the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = e$ about the y-axis.

a) 2π
b) 1
c) $2\pi(e-1)$
d) $e-1$
e) none of the above

15. Find the point of inflection of $r(x) = \frac{x-2}{(x+1)^2}$.

a) $(2,0)$
b) $\left(5, \frac{1}{12}\right)$
c) $\left(8, \frac{2}{27}\right)$
d) $\left(4, \frac{2}{25}\right)$
e) none of the above

16. If f

18. If x and y are real numbers such that $x^2 + y^2 = 8$, what is the maximum possible value of $x \quad y$

22. Determine which function would produce the greatest area between the function and $g(x)=0$ from $x=1$ to $x=100$.

a) $f(x)=x^{10}$

b) $f(x)=10^{x}$

c) $f(x)=10x$

d) $f(x)=\log_{10}(x^{10})$

e) none of the above

Reminder

Question 23 will be used again as a tie-breaker, if necessary.

23. Consider the particle traveling clockwise on the elliptical path $\frac{x^2}{100}+\frac{y^2}{25}=1$. The particle leaves the orbit at the point $(-8,3)$ and travels in a straight line tangent to the ellipse. At what point will the particle cross the y-axis?

a) $\left(0,\frac{25}{3}\right)$

b) $\left(0,-\frac{25}{3}\right)$

c) $(0,9)$

d) $\left(0,\frac{7}{3}\right)$

e) none of the above

24. If the tangent (DDE!6I6I6,6! l :6 n o q o(TEEEEE6I6I6(TEEEEE6I6ETkk?T9,E6I6I6 c m6B ToSR,:6,,,/E!9/

28. Find the 151^{st} derivative of $f(x) = \sin(-x)$.

a) $-\cos(-x)$

b) $\sin(-x)$

c)

32. Which of the following definite integrals has a positive value?

a) $\int_{0}^{\frac{2\pi}{3}} \sin(3x+\pi)\,dx$

b) $\int_{\frac{2\pi}{3}}^{0} \sin(3x+\pi)\,dx$

c) $\int_{\frac{-3\pi}{2}}^{0} \sin(3x+\pi)\,dx$

d) $\int_{0}^{\frac{-3\pi}{2}} \sin(3x+\pi)\,dx$

e) none of the above

35. Find: $\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

a) $-\frac{1}{4}\operatorname{arcsec}\left(\frac{x}{4}\right)+C$

b) $\frac{1}{4}\operatorname{arcsec}\left(\frac{x}{4}\right)+C$

c) $-\frac{\sqrt{16-x^2}}{16x}+C$

d) $\frac{\sqrt{16-x^2}}{16x}+C$

e) none of the above

36. The derivative of $f(x)=5x^x$ is

a) $5xx^{x-1}$

b) $5x^x \ln x$

c) $\frac{5x^x}{\ln x}$

d) $5x^x(1+\ln x)$

e) none of the above

37. How much work is done by a colony of ants in building a conical ant hill with height and diameter of the base both 1 ft, using sand initially at ground level and with a density of 150 lb/ft^3?

a) $\frac{75}{8}\pi$ ft-lb

b) $\frac{25}{2}\pi$ ft-lb

c) $\frac{25}{8}\pi$ ft-lb

d) 25π ft-lb

e) none of the above

38. Air is escaping from a spherical balloon at the constant rate of 200π cm^3/s. What is the radius of the balloon when its radius is decreasing at 2 cm/s?

a) 5 cm
b) $5\sqrt{2}$ cm
c) 10 cm
d)