- 3. A designer wants to introduce a new line of bookcases. He wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is C(x). Assume that C(x) is a differentiable function. Which of the following must be do to find the minimum average cost,  $c(x) = \frac{C(x)}{x}$ ?
  - (I) Find the points where c'(x) = 0 and evaluate

- 6. A particle is moving in the first quadrant downward on the hyperbola  $\frac{x^2}{16} \frac{y^2}{64} = 1$ . It leaves the hyperbolic path at the point (5,6) and continues along a straight line. At what point does the particle cross the x-axis?
  - a)  $\frac{16}{-},0$

9. Calculate the derivative of the function  $f(x) = x^{(e^x)}$ .

a) 
$$e^x x^{\left(e^x-1\right)}$$

b) 
$$x^{(e^x)} \left[ e^x \ln x + \frac{e^x}{x} \right]$$

c) 
$$x^{(e^x-1)}e^x\left[\frac{1}{x}+\ln x\right]$$

d) 
$$e^x x^{\left(e^x-1\right)} \ln x$$

e) none of the above

12. A hemispherical bowl of radius a contains water to depth h. Find the volume of the water in the bowl.

14. If f and g are both differentiable and  $h=f \circ g$ , h'(2) equals

- a)  $f'(2) \circ g'(2)$
- b) f'(2)g'(2)
- c) f'(g(x))g'(2)
- d) f'(g(2))g'(2)
- e) none of the above

15. Let f(x) = x |x|. Find f'(0).

- a) 0
- b) 1
- c) -1
- d) does not exist
- e) none of the above

16. Evaluate:  $\lim_{x \to 0} \frac{1 - e^{-x}}{\sin x}$ 

- a) ∞
- b) 1
- c)  $\sqrt{3}$
- d)

18. Evaluate: 
$$\int_{0}^{-1} \frac{1}{\sqrt{x+1+\sqrt{x+1}}} dx$$

a) 
$$-2\sqrt{2} + \ln\left(3 + 2\sqrt{2}\right)$$

b) 
$$\frac{1}{-2\sqrt{2}} + \ln\left(3 + 2\sqrt{2}\right)$$

c) 
$$-2\sqrt{2} + \ln\left(2 + 3\sqrt{3}\right)$$

$$d) \qquad \frac{1}{-2\sqrt{2}} + \ln\left(2 + 3\sqrt{3}\right)$$

- e) none of the above
- 19. Find the length of the curve  $x = \frac{y^4}{4} + \frac{1}{8y^2}$  from y = 1 to y = 2.
  - a) —

21. Evaluate: 
$$\int_{0}^{1} \sqrt{\frac{1+x}{1-x}} dx$$

a) 
$$\pi - 1$$

b) 
$$\frac{\pi}{2} - 1$$

c) 
$$\frac{\pi}{2} + 1$$

d) 
$$\frac{\pi}{4} + 1$$

e) none of the above

22. If 
$$f(x) = \ln |Cx|$$
, for  $C \neq 0$ , then  $f'(x) =$ 

a) 
$$\frac{1}{|x|}$$

b) 
$$\frac{1}{|Cx|}$$

c) 
$$\frac{1}{x}$$

d) 
$$\frac{1}{Cx}$$

e) none of the above

23. If a trigonometric substitution in terms of a secant function in the variable 
$$\theta$$
 is used to

solve 
$$\int_{\frac{5}{2}}^{\frac{5}{\sqrt{3}}} \sqrt{4x^2 - 25} \ dx$$

- What is the area of the largest rectangle that can be inscribed in the region bounded by  $y = 3 x^2$  and the x-axis? 24.
  - a)
  - b)
  - $\begin{array}{c}
    4 \\
    6 \\
    3\pi \\
    \hline
    2 \\
    \sqrt{5}
    \end{array}$ c)
  - d)

- 26. A ball is thrown straight up from the ground. How high will it go? Assume that g is the absolute value of the gravitational acceleration and  $\,v_{0}$  is the initial velocity.
  - $gv_0^2$ a)
  - b)  $\frac{1}{2}g^2 + gv_0$
  - c)  $\frac{1}{2}v_0 + \frac{1}{2}v_0^2 g$
  - $\frac{1}{2}v_0^2g^{-1}$ d)
  - none of the above e)
- Given  $F(x) = \int_{0}^{x^2} e^{5t-t^2} dt$ , find F'(2). 27.
  - a)
  - b)
  - $4e^{4}$   $-3e^{4}$   $-3e^{4} 5$ c)
  - $\frac{1}{5}e^4 e^{10}$ d)
  - none of the above e)
- 28. Two electrons repel each other with a force inve

29. If 
$$f(x) = \sqrt{x} - x + 9$$
, for  $x \ge \frac{1}{2}$ , and  $g = f^{-1}$ , then  $g'(9)$  is

- a) -2
- b)  $-\frac{5}{6}$
- c)  $-\frac{6}{5}$
- d) -1
- e) none of the above

30. Integrate: 
$$\int x \ln(x^2) dx$$
.

a) 
$$\frac{x^2 \ln(x^2)}{2} - \frac{x^3}{3} + C$$

b) 
$$\frac{x^2 \ln(x^2)}{2} - \frac{x^2}{2} + C$$

c) 
$$\frac{x^2 \ln(x^2)}{2} + \frac{x^2}{2} + C$$

$$d) \qquad \frac{x^2 \ln x}{2} - \frac{x}{2} + C$$

31. Evaluate: 
$$\lim_{x \to \sqrt[3]{2}} \left( \frac{x^2}{2} - \frac{1}{x} \right)$$

b) 
$$\frac{2}{3} \cdot \frac{1}{\sqrt[3]{2}}$$

$$d) \qquad \frac{3}{2} \cdot \sqrt[3]{2} - \frac{1}{\sqrt[3]{2}}$$

- 32. If the product function  $h(x) = f(x) \cdot g(x)$  is continuous at x = 0, then the following must be true about the functions f and g. (Choose just one answer.)
  - a) Both functions must be continuous at x = 0.
  - b) One of them must be continuous at x = 0, but not necessarily the other.
  - c) Both must be discontinuous at x = 0.
  - d) They may be continuous or not at x = 0, all options are possible.
  - e) none of the above
- 33. One way to compute  $\frac{1}{2}$  the area of the unit circle is to integrate

34. Evaluate:  $\int_{-1}^{1} \frac{dx}{x^2 - 6x + 9}$ 

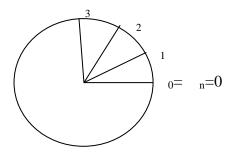
37. How many zeros does the function  $f(x) = x^4 + 3x + 1$  have in the interval [-2, -1]?

- a) no zeros
- b) exactly one zero
- c) exactly two zeros
- d) exactly three zeros
- e) none of the above

38. Given that  $f(n) = \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n}\sqrt{n+n}}$ . Find  $\lim_{n \to \infty} f(n)$ .

- a)  $1 + \frac{1}{e}$
- b)  $\sqrt{2}$
- c) 1-<u>e</u>
- d)  $\frac{e}{2\sqrt{2}}$

40. We cut a circular disk of radius r into n circular sectors as shown in the figure, by marking the angles  $\theta_i$  at which we make the cuts ( $\theta_0 = \theta_n$  can be considered to be the angle 0). A circular sector between two angles  $\theta_i$  and  $\theta_{i+1}$  has an area  $\frac{1}{2} r^2 \Delta \theta_i$ , where  $\Delta \theta_i = \theta_{i+1} - \theta_i$ .



We let  $A_n = \sum_{i=0}^{n-1} \frac{1}{2} r^2 \Delta \theta_i$ . Then the area of the disk, A, is given by:

- a)  $A_n$ , independent of how many sectors we cut the disk into
- b)  $\lim_{n\to\infty} A_n$
- c)  $\int_{0}^{2\pi} \frac{1}{2} r^{2} d\theta$
- d) all of the above
- e) none of the above