University of North Georgia
Nineteenth Annual Sophomore Level Mathematics Tournament

You may write in this test booklet. Only the el

3. Find the constant c

6. Let f be a continuous function such that f is strictly increasing, $f(0)=0,$ and $f(2)=4.$ Let g be the inverse of $f.$ Evaluate $\int_0^2 f(x)\,dx+\int_0^4 g(y)\,dy.$

a) 6
b) 8
c) 12
d) 16
e) None of the above

7. A cable hangs in the form of a catenary between two towers 200 feet apart. The hanging cable is modeled by the equation $y=150\cosh\dfrac{x}{150}$. One possible integral that represents the arc-length along the cable between the two towers is:

a) $\int_{-100}^{100}\sinh\dfrac{x}{150}\,dx$
b) $\int_{-100}^{100}\cosh^2\dfrac{x}{150}\,dx$
c) $\int_{-100}^{100}\sinh^2\dfrac{x}{150}\,dx$
d) $\int_{-100}^{100}\cosh\dfrac{x}{150}\,dx$
e) None of the above

8. Find a if $\lim\limits_{x\to 1}\dfrac{x+1}{x^2+ax+1}=\dfrac{1}{9}.$

a) 2
b) 4
c) 8
d) 16
e) None of the above

9. Find the integral: $\int_0^{\infty} \frac{1}{x^3+1}\,dx$.

a) $-\frac{\pi\sqrt{3}}{9}$

b) $\frac{2\pi\sqrt{3}}{9}$

c) $\frac{15\pi}{\sqrt{3}}$

d) $\frac{3\pi\sqrt{2}}{2}$

e) None of the above

10. Find $\left(f^{-1}\right)'(1)$, where $f(x) = x^3 - 3x^2 + 21$ for $x < 0$.

a) 12

b) $\frac{1}{24}$

c) $-\frac{1}{3}$

d) Undefined, since f^{-1} is undefined for $x < 0$.

e) None of the above

11. Evaluate $\frac{d}{dx}\int_{-\pi}^{x} \sin^2 t \cos^7 t\,dt$ at $x = \pi$.

a) -1

b) 0

c) 1

d) 2

e) None of the above

12. Find the definite integral: $\int_2^4 \frac{x^2+1}{(2x-3)^2}\,dx$.

a) $\frac{9}{4}-\frac{3}{5}\ln 5$

b) $\frac{9}{5}-\frac{3}{4}\ln 5$

c) $\frac{9}{5}\ln 5+\frac{3}{4}$

d) $\frac{9}{5}+\frac{3}{4}\ln 5$

e) None of the above

13. Which of the following functions are concave upward on an open interval containing $x=0$?

i) $\ln x$ ii) x^2 iii) $\cos x$ iv) $\frac{1}{x^2-1}$ v) $\tan x$

a) Only i

b) Only ii

c) i and iii

d) i and iv

e) None of the above

14. Find the limit: $\lim_{h\to 0} \frac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$.

a) 3

b) $2\sqrt{2}$

c) 1

d) 0

e) None of the above

15. Four feet of wire is available to form a square or a circle or both. How much of the wire should be used for the square and how much should be used for the circle to ensure the maximum total area?

    a) 3 feet should be used for the circle and 1 foot for the square.
    b)

18. Determine a and b in the formula $\sin x + \cos x = a\sin(x+b)$ and evaluate the integral $\dfrac{1}{\sin x + \cos x}\,dx$.

a) $-\dfrac{1}{\sqrt{2}}\ln\left|\csc\left(x+\dfrac{\pi}{4}\right)+\cot\left(x+\dfrac{\pi}{4}\right)\right|+C$

b) $\dfrac{\sqrt{3}}{2}\ln\left|\sec\left(x+\dfrac{\pi}{3}\right)+\tan\left(x+\dfrac{\pi}{3}\right)\right|+C$

c) $-\dfrac{\sqrt{3}}{2}\ln\left|\csc\left(x+\dfrac{\pi}{3}\right)+\cot\left(x+\dfrac{\pi}{3}\right)\right|+C$

d) $\dfrac{1}{\sqrt{2}}\ln\left|\sec\left(x+\dfrac{\pi}{4}\right)+\tan\left(x+\dfrac{\pi}{4}\right)\right|+C$

21. Find the equation of the tangent line to the graph of $f(x)=\dfrac{3-\dfrac{1}{x}}{x+5}$ at $(-1,1)$.

a) $y=1$

b) $y=x-1$

c) $x+y=1$

d) $y=\dfrac{3}{5}x+1$

e) None of the above

22. Find $\dfrac{f'(1)}{f(1)}$ if $\lim\limits_{x\to 1}\dfrac{f(x)-2}{x^2-1}=3,$ where $f(x)$

24. Given that two non-negative numbers have a sum of 9 and the product of one number and the square of the other number is a

28. Find the dimensions of the rectangle of largest area which can be inscribed in the closed region bounded by the *x*-axis, *y*-axis and the graph of $y = 8 - x^3$.

a) $2^{1/3}$ by 6
b) $3^{1/2}$ by 6
c) $6^{1/2}$ by 3
d) $2^{1/3}$ by 3
e) None of the above

29. Find the integral: $\dfrac{x^2 dx}{(1+x)^{1/3}}$.

a) $\dfrac{3}{20}(x+1)^{1/3}\left(10-5x+3x^2\right)+C$

b) $\dfrac{1}{3}(x+1)^{1/3}\left(9x-5x^2+8x^3\right)+C$

c) $\dfrac{3}{40}(x+1)^{2/3}\left(9-6x+5x^2\right)+C$

d) $\dfrac{1}{3}(x+1)^{2/3}\left(10x-6x^2+3x^3\right)+C$

e) None of the above

Reminder

Question 30 will be used as a tie-breaker, if necessary.

30. Find the definite integral: $\int_0^1 e^{\sin^2 x} e^{\cos^2 x}\, dx$.

a) 0
b) e
c) π
d) 1
e) None of the above

31.

34. Find the definite integral: $\int_1^e (\ln x)^2\,dx.$

a) $e-2$
b) e
c) 1
d) $\frac{e}{4}$
e) None of the above

35. How many functions h are there such that $h'(x)=h(x)$?

a) 2
b) 3
c) 4
d) Infinitely many
e) None of the above

36. If $\cos y = x,$ find $\frac{dy}{dx}.$

a) $\sqrt{1-x^2}$
b) $\sec^2 y$
c) $-\csc y$
d) $\frac{1}{1+x^2}$
e) None of the above

37. If $y=\log_2\left(x^3\right),$ find $\frac{dy}{dx}.$

a) $3\log_2\left(x^2\right)$
b) $\frac{3}{x\ln 2}$
c) $\frac{x\ln 3}{2}$
d) $\frac{6}{\ln x}$
e) None of the above

38. Differentiate $y = \sec^2\left(x^4\right)\tan^3\left(x^4\right)$.

a) $4x^3\sec^2\left(x^4\right)\tan^2\left(x^4\right)\ \ 3\sec^2\left(x^4\right)+2\tan^2\left(x^4\right)$

b) $4x^4\sec^2\left(x^4\right)\tan^2\left(x^4\right)\ \ 3\sec^2\left(x^4\right)+2\tan^2\left(x^4\right)$

c) $4x^3\sec^2\left(x^3\right)\tan^2\left(x^4\right)\ \ 3\sec^2\left(x^4\right)+2\tan^2\left(x^4\right)$

d) $4x^3\sec^2\left(x^4\right)\tan^2\left(x^3\right)\ \ 3\sec^2\left(x^4\right)+2\tan^2\left(x^4\right)$

e) None of the above

39. Find the volume of the solid obtained by revolving the region in the first quadrant bounded by the *x*-axis, the *y*-axis, and the line $4x+2y=8$ about the *x*-axis.

a) 16π

b) $\dfrac{32\pi}{3}$

c) $\dfrac{817}{10}$

d) $3\pi\sqrt{2}$

e) None of the above

40. An article in the Wall Street Journal's "Heard on the Street" column (Money and Investment, August 1, 2001) reported that investors often look at the "change in the rate of change" to help them "get into the market before any big rallies." Your stock broker alerts you that the rate of change in a stock's price is increasing. As a result you

a) Can conclude the stock's price is decreasing
b) Can conclude the stock's price is increasing
c) Cannot determine whether the stock's price is increasing or decreasing
d) Can conclude the stock's price is neither increasing or decreasing
e) None of the above