University of North Georgia

Sophomore Level Mathematics Tournament

April 5, 2014

Solutions for the Afternoon Team Competition

Round 1

Volume = $r^2h = 6^2 \cdot 3 = 6 \cdot 6 \cdot 3 = 6 \cdot 2 \cdot 3 \cdot 3 = 12 \cdot 9$

The answer is 12 pieces.

Round 2

We think about the complement people choose different numbers.

The first person can choose any number (positive integer less than 11: from 1 to 10), then the second person would have 9 (different) numbers to choose (9/10), the third person 8 (different) numbers to choose, etc. So the probability that the 4 people choose different numbers is:

$1 \cdot \frac{9}{10} \cdot \frac{8}{10} \cdot \frac{7}{10} = \frac{504}{1000}$. Hence the probability that two of the people choose the same number is:

$1 - \frac{504}{1000} = \frac{496}{1000} = 0.496$.

Round 3

Since $f(x)$ is divisible by $(x-1)^3$, $x^4 + ax^2 + bx + c = (x-1)^3(x+d)$ for some real number d.

Now if we equate the coefficient of x^3 on both sides we see that $d = 3$.

Then $f(2) = (2-1)^3(2+3) = 5$.

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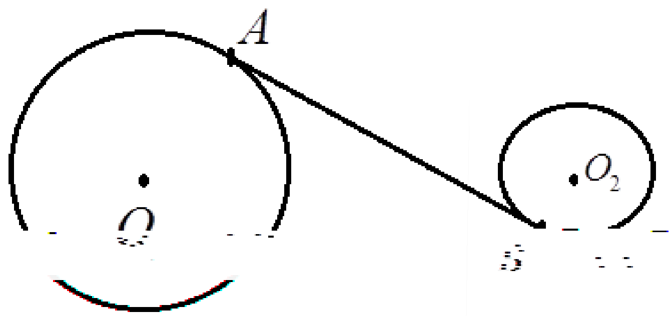
Round 4



We get 16 and 8 from the fact that the triangles are congruent. Then we use the Pythagorean Theorem twice getting $a \quad \sqrt{220} \quad 2\sqrt{55}$ and $b \quad \sqrt{55}$. So $a \quad b \quad 3\sqrt{55}$.

Round 5

We have $\cot \quad \cot \quad 4$, so $\frac{1}{\tan} \quad \frac{1}{\tan} \quad 4$ and $\frac{\tan \quad \tan}{\tan \quad \tan} \quad 4$.

Thus, $\tan \quad \tan \quad \frac{\tan \quad \tan}{4} \quad \frac{7}{4}$.

Then

Round 6

Let r be the radius in inches. Then the area in square inches is $\quad r^2$ which must be a natural number according to the problem.

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Round 7

$f(1) = 2+4+6+\cdots+100 = (2+4+6+\cdots+98)+100$ and

$g(1) = 1+3+5+\cdots+99 = 1+(2+1)+(4+1)+\cdots+(98+1) = 50+(2+4+6+\cdots+98)$

The sum $2+4+6+\cdots+98$ can be evaluated as $2(1+2+3+\cdots+49) = 49 \cdot 50 = 2450$.

Consequently, $f(1) = 2450+100 = 2550$ and $g(1) = 50+2450 = 2500$.

So $f^2(1) - g^2(1) = (f(1)+g(1))(f(1)-g(1)) = (2550+2500)(2550-2500) = 252{,}500$

Dividing 252,500 by 100 gives 2525.

Round 8

We are looking for $abcd \le 1200$, where *a*, *b*, *c*, and *d* are primes with $a < b < c < d$. We solve this problem by finding the largest possible value for *a*, then for *b*, and so on. It turns out you can find the answer by making a dozen or so calculations.

1. Establish a benchmark by multiplying consecutive primes:

$$2 \cdot 3 \cdot 5 \cdot 7 = 210$$
$$3 \cdot 5 \cdot 7 \cdot 11 = 1155$$
$$5 \cdot 7 \cdot 11 \cdot 13 = 5005$$

which is the smallest value of *abcd* where $a > 3$

Round 9

Note that paths cannot be repeated. We will count all the possible paths from *S* to *F* that pass through *M* or *N* separately and then subtract any paths that are repeated. This is known as an inclusion-exclusion method.

Part 1: Paths from *S* to *F* through *M* (or simply *SMF* paths) these go from *S* to *M* and then to *F*. There are exactly 3 paths from *S* to *M* (of length 3 each). There are exactly 10 paths from *M* to *F* (of length 5 each). For each of the 2 *SM* paths, there are 10 *MF* paths giving a total of 3 10 30 *SMF* paths.

Part 2: Paths from *S* to *F* through *N* (or simply *SNF* paths) these go from *S* to *N* and then to *F*. There are 15 paths from *S* to *N* (of length 6 each). There are 2 paths from *N* to *F* (of length 2 each). For each of the 15 *SN* paths, there are 2 *NF* paths giving a tot