Twenty Second Annual University of North Georgia Mathematics Tournament

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

1. Find the integral: $\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin x}\, dx$

   a) 4

   b) $\sqrt{1}$

   c) 2

   d)

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3. Find the maximum value of $y \quad 6\cos x \quad 14x \quad 5$ on $\frac{3}{2}, 0$.

a) 1

b) 3

c) 5

d) 7

e) None of the above

4. If g is the inverse function of $f(x) \quad 2x \quad \ln x$, find $g(2)$.

a) $\frac{1}{3}$

b) $2 \quad e^2$

c) $\frac{3}{2}$

d) $\frac{1}{2}$

e) None of the above

5. Find the definite integral: $\int_0^1 \sqrt{x \quad x^2}\, dx$

6. If f is continuous, then $\int_0^1 f(1-x)\,dx$ is equal to

   a) $\int_0^1 f(x)\,dx$

   b) $\int_0^1 f(x)\,dx$

   c) $\int_0^1 f(-x)\,dx$

   d) $\int_0^1 f(-x)\,dx$

   e) None of the above

7. Find $f(t)$ so that $\int f(t)\,dt = \frac{t}{2}\sqrt{4-t^2} - 2\arcsin\frac{t}{2}$.

   a) $\frac{1}{\sqrt{1-\frac{t^2}{4}}} + C$

   b) $\sqrt{4-t^2} + C$

   c) $\arcsin\frac{t}{2} + C$

   d) $t\sqrt{4-t^2} + C$

   e) None of the above

8. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

   a) $4^{2/3}$

   b)

9. Find the definite integral: $\int_0^1 \arcsin x \, dx$

a) 1

b) $\frac{}{2}$ 1

c) $\frac{}{2}$

d) $\frac{}{3}$

e) None of the above

10. Suppose that $\int_0^x f(t)\,dt \quad \sin x$, find $f(\ \ )$.

a) 1

b) 1

c) 0

d) $\frac{\sqrt{\ }}{}$

e) None of the above

11. Evaluate the limit: $\lim_{x} \frac{\frac{}{2} \arctan x}{\ln \frac{2x \ 1}{1 \ 2x}}$.

a)

12. A light house is located on a small island, 3 km away from the nearest point *P* on a straight shoreline, and its light makes four revolutions per minute. How fast is the beam light moving along the shoreline when it is 1 km from *P*?

a) 1600 km/h
b) 480 km/h
c) 1800 km/h
d) 7200 km/h
e) None of the above

13. If f is continuous and $\int_0^2 f(x)\,dx = 6$, evaluate $\int_0^{\frac{\pi}{2}} f(2\sin\theta)\cos\theta\,d\theta$.

a) 3
b) 6
c) 2
d) 12
e) None of the above

14. Assume that f is continuous and that $f(1) = 3$, $f'(1) = 2$, and $\int_0^1 f(x)\,dx = 5$.

Find $\int_0^1 x^2 f'(x)\,dx$.

a) 6
b) 1
c) $\frac{1}{2}$
d) 2
e) None of the above

17. Evaluate the integral: $\int \frac{dx}{1 \quad e^{x}}$

a) $x \quad 1 \quad C$

b) $x \quad \ln e^{x} \quad C$

c) $x \quad \ln e^{x} \quad C$

d) $x \quad \ln \quad e^{x} \quad 1 \quad C$

e) None of the above

18. Water is poured into a conical cup at the rate of $\frac{2}{3}$ cubic inches per second. If the cup is 6 inches tall and if the top of the cup has a radius of 2 inches, how fast is the water level rising when the water is 4 inches deep?

a) $\frac{3}{8}$ in/sec

b) $\frac{3}{8}$ in/sec

c) $\frac{1}{4}$ in/sec

d) in/sec

e) None of the above

19. Which of the following expressions equals to ?

a) 0

b) $(\quad 1)^{n} \frac{n!}{x^{n}}$

c)a) (1)^{n} $\frac{(n \quad 1)!}{x^{n} \quad 1}$ 0 0n8.D 45 -0BCm/GS14 gs2 Tc[(a)-BTTf1 0 0 1 126.02 223.73 Tm[5)] TJET EMC /P<</MCID 1] T74

20. Let $f(k) \quad \frac{d^j}{dx^j} \; e^{kx}$. Find $f\ (k)$.

a) $k^{j\ 1}e^{kx}\ \ k^2 \quad j$

b) $k^{j\ 1}e^{kx}\ \ k \quad j^2$

c) $k^{j\ 1}e^{kx}\ \ kx \quad j$

d) $k^{j\ 1}e^{kx}\ \ x \quad jk$

e) None of the above

21. Find the definite integral:

$\sqrt{\quad}$

a) $\frac{6}{5}$

b) $\frac{4}{9}$

c) $\frac{3}{2}$

d) $\frac{1}{3}$

e) None of the above

22. The sequence of numbers $\frac{2}{1}$, $\frac{3}{2}^{\ 2}$, $\frac{4}{3}^{\ 3}$,…, $\frac{101}{100}^{\ 100}$,… gets as close as you want to:

d4. EMC /P<a))] T8>BDC /GS1gs1E0.8C /GS (792 B1 0 0 1 9E7>BDC B/F2 s a402 330.77 Tm[( )] TJEo9>BDC B/F82Go9>1E3)] TJET

23. What is the minimum vertical distance between the parabolas $y \quad x^2 \quad 1$ and $y \quad x \quad x^2$.

a) $\frac{7}{8}$

b) $\frac{5}{8}$

c) $\frac{9}{8}$

d) $\frac{1}{8}$

e) None of the above

24. If the radius of the circle increases from r_1 to r_2, the average rate of change of the area of the circle with respect to the radius is

a) Greater than $2 \quad r_2$

b) Less than $2 \quad r_1$

c) Equal to $2 \quad \frac{r_1 \quad r_2}{2}$

d) Equal to

e) None of the above

25. Find the limit: $\lim_{x} \quad \sqrt{x^2 \quad 4x} \quad x$

a) 1

b) 2

c) 4

d) Does not exist

e) None of the above

29. Suppose you have two linear functions f and g shown below.

Then $\lim_{x \to a} \frac{f(x)}{g(x)}$

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37. Find the length of the curve with the equation $x^{2/3} \quad y^{2/3} \quad 1$

a) 6
b) 4
c) 24
d) 12
e) None of the above

38. In the figure below, find the dimension of the rectangle with maximal area in the $45 \; -45 \; -90$ right triangle with legs of length 1.

a) $\sqrt{2}, \sqrt{2}$
b) $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{4}$.
c) $\frac{1}{2}, \frac{1}{4}$

39. Find the derivative $\frac{df}{dx}$ of the function $f(x)$ $x^{\sin x}$.

a) $\sin x \; x^{\sin x \; 1}$

b) $\sin x \; x^{\cos x}$

c) $^{\sin x}$ cos ln $\frac{\sin x}{}$

d) $x^{\sin x} \ln \sin x$

e) None of the above

40. Suppose $f(x)$ is differentiable everywhere and $f(x)$ $2f(\ x)$ $\sin x$ for all real x.

What is the value of f $\frac{}{4}$?

a) 1

b) 1

c) $\frac{\sqrt{2}}{2}$

d) $\frac{\sqrt{2}}{2}$

e) None of the above