

Twenty-Fifth Annual  
University of North Georgia  
Mathematics Tournament

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Mathematics Tournament

You may write in this test booklet. Only the el

)LQG WKH <sup>WDQ</sup>3QWBJUDO  
x

D  $\frac{WDQ}{x}$  C

E  $\frac{WDQ}{x}$  C

F x WDQ |  $\frac{WDQ}{x}$  C

G x WDQ x OQ  $\frac{WDQ}{x}$  C

H 1RQH RI WKH DERYH

7KH UDGLXV RI D ~~HSKRUH~~ HLV PPH, ~~MLVPHU~~ HFW WR  
— PP HVWLPDWH WKHLU ~~QURSD~~ JDRXG ~~HLU~~ WKH VSKHUH  
/HDYH \RXU DQVZHU LQ WHUPV RI

D  $\frac{S}{x}$

E  $\frac{S}{x}$

F S

G S

H 1RQH RI WKH DERYH

,I \RX QHHG WKLV GRFXPHQW LQ DQRWKHU JIRGRU ISODD VH HPD

)LQG DQ HTX DQWLRQWRO LVKHWKWK JDWD SK SRLQW ZKHU  
WKH WDQJHQW OLQZLW ENHW SDQWQWZWKDQ  
DQG WKH SRLQW OLHV LQ WKH ILUVW TXDGUDQW

D x y

E x y

F x y

G x y

H 1RQH RI WKH DERYH

7KH IXQFWLRQ KDV DQ LQYHUHQXQFWLRQ

D -

E -

F -

G -

H 1RQH RI WKH DERYH

:KDW LV WKH DYFUDHWDZRRHDDQG DPHVHWZ"HHQ

D S P

E <sup>S</sup> P

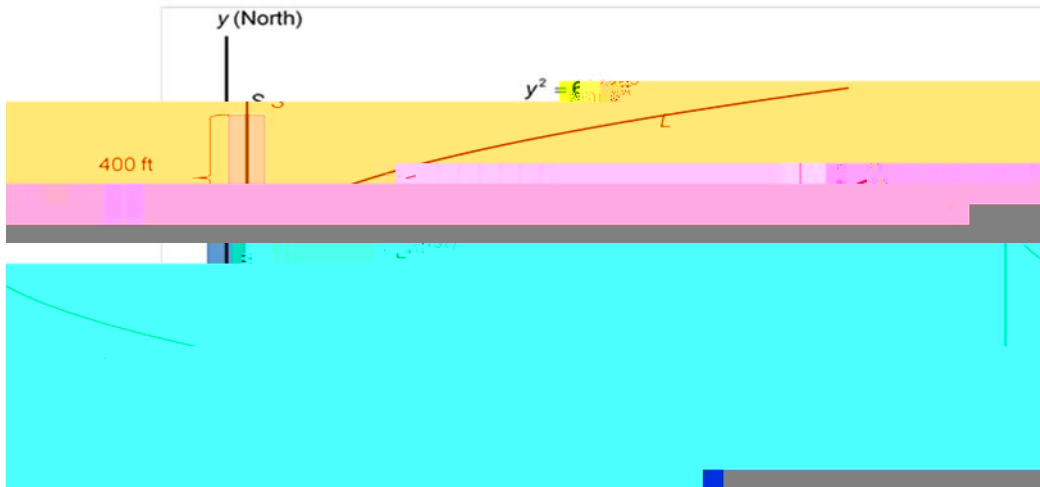
F <sup>S</sup> P

G <sup>S</sup> P

H 1RQH RI WKH DERYH

,I \RX QHGH WKLV GRFXPHQW LQ DQRWKHQJIRGRWU SDGDVH HPD

\$ UDLOURDG WUDLQ LV PRYLQJ DW DW SHRGGI R7KH PLUID F  
 WKH VKDSH R<sub>y</sub> WKH S<sub>y</sub>UDKERZOD , I W KHL WIXQJ LLQ MXXH HDV  
 KRZ IDVW VSKHI WKH G<sub>z</sub> ER PRRWLLYHU DORQJ WKH ZDOO DW  
 UHDFKHV WKH HQG RI WKH ZDOO



D PL K  
 E PL K  
 F PL K  
 G PL K  
 H 1RQH RI WKH DERYH

7ZR SODQH V DW WKUH VDSFU D DFKLLVXUHQD WKH SRRUWW KR Q E C  
 IURP WKH ZHVW 7KH SODQH IURP GWKVI QPULW RRP WKHL Q  
 DLUSRUW 7KH SODQH IURP WKH ZPHWURP WOKHQD LDVSRUW  
 IDVW DUH WKH SODEK R V S W R D W K W Q D W L Q V W D Q W "

D PSK  
 E PSK  
 F PSK  
 G PSK  
 H 1RQH RI WKH DERYH

, I \RX QHHG WKLV GURFXUPLQDW ISO HDQRYH #FDL OI FX QVX FDOO

)LQG WKH3LQW HFDUO

D — VLQ — xLQ<sup>x</sup> C

E — VLQ — xV<sup>x</sup>EQC

F — VLQ — xV<sup>x</sup>EQC

G — VLQ — xLQ<sup>x</sup> C

H 1RQH RI WKH DERYH

8VH WKH &KDLQ 5XOH DQG WKH 6HF006 WRQLDGHQWDO

,I \RX QHHG WKLV GRFXPHQW LQ DQRWKXQJIRGRDW ISDQDVH HPD

)LQG WKHL<sub>x of</sub> LPL<sup>x</sup>W

D

E

F

G e

H 1RQH RI WKH DERYH

7KH VHW Ry SLQOMV SODxQHy|VDWLLWVLQJFXUYH HQFORV  
D UHJLRQ &RPSXWH LWV DUHD

D —

E —

F

,I \RX QHHG WKLV GRFXPHQW LQ DQRW#XHQJIRGRDW ISDQDVH HPD

(YDOXDWH WKH ~~SHLDL~~<sup>X</sup> LQWH LQWHJUDO  
X

D  $\sqrt{OQ}$

E  $\sqrt{OQ}$

F -

G  $\sqrt{DUFWD:Q}$   $\sqrt{S}$  .  
© 31

H 1RQH RI WKH DERYH

(YDOXDWH WKH ~~SHLDL~~<sup>S</sup> LQWH LQWHJUDO  
S

D FRV VLQ

E S

F

G VLQ FRV

H 1RQH RI WKH DERYH

)LQG WKH ~~SHLDL~~<sup>x</sup> ~~SHLDL~~<sup>VLQ</sup>  
x<sup>o</sup> f  $\sqrt{x}$  x

D f

E

F

G

H 1RQH RI WKH DERYH





s

/HW EH WKH FRQWLQXR X<sup>3</sup> W~~DE~~ WL~~DO~~ Q~~3~~  $\frac{x^f x}{x}$  dx

ZKDW LV WK~~H~~ Y~~D~~ OXH RI

D

E

F

G

H 1RQH RI WKH DERYH

/HW WKH f I X<sup>3</sup> W~~DE~~ WL~~DO~~ Q~~3~~ W~~DE~~ Q~~3~~ G

/HfWEH D IXQE WDLR Q QWGD QW V X<sup>x</sup> F<sup>f</sup> K<sup>t</sup> W<sup>t</sup> KDW  
:KDW LV WKH<sup>c</sup> Y<sup>c</sup> DOXH RI

D -

E -

F -

G

H 1RQH RI WKH DERYH

(YDOXDWH WKH G<sup>f</sup> H<sup>dx</sup> I<sup>dx</sup> L<sup>dx</sup> Q<sup>dx</sup> W<sup>dx</sup> LQWHJUDO  
x<sup>dx</sup> x<sup>dx</sup>

D - OQ

E e

F f

G OQ

H 1RQH RI WKH DERYH

/HMEH WKH IXQFWLRQ ZILMWHQ/EN WUKDISR @ DRZLQJ

/HMVx f x x :KLFK RI WKH IROORZLQJ LV WUXH"

D h h h !

E h h h

F h h ! h !

G h h! h !

H 1RQH RI WKH DERYH

8VH WKH yDXFW WKDWR HYD<sup>f</sup>OXDXWH

D -

E -

F -

G

H 1RQH RI WKH DERYH

,I \RX QHHG WKLV GRFXPHQW LQ DQRW#XHQJIRGRDWU ISDQDVH HPD

5HPLQGHU

4XHVWLRQ ZLOO EH XVHG DV D WLH EUHDNHU

)LQG WKH YROXPHWDLWKH ERORQDORQH GHV UHJLRQ H

x x y DERXW xWKH OLQH

D <sup>S</sup>-OQ

E SOQ

F S

G S O Q

H 1RQH RI WKH DERYH

6XSSRVH D GLIHXUHQWLRQDORQH ERORQH GHV ,I

f ZKDW LV WKH ODUJHVW SRVVLEOH YDOXH RI

D

E

F

G

H 1RQH RI WKH DERYH

6XSSRVH WKDW RQ WKH LQWHUYDO

ø

,I \RX QHGG WKLV GRFXPHQW LQ DQRWKHUJIRGRDW ISODD VH HPD

,I \RX QHHG WKLV GRFXPHQW LQ DQRWKHU IRUPDW SOHDVH HPD



6XSSfRX/H f x f y x y xy IRU DOO UHYDQOXPEHUV  
x<sup>o</sup> x

)LQfG

D

E

F

G

H 1RQH RI WKH DERYH

7KH SRLQW RQ WKH FXUYH

,I \RX QHHG WKLV GRFXPHQW LQ DQRWKXIJIRGRDW ISDHO VH HPD



